

# An Information-Theoretic Detector Based Scheme for Registration of Speckled Medical Images

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**Abstract**—Several studies dealt with medical ultrasound registration. Their similarity metrics relied on pixel-to-pixel intensity comparisons. Hence, they are not well suited to the case of speckled images. To better handle the speckle noise, our previous work proposed an information-theoretic feature detector-based registration approach. This work aims to extend it to the cases where the image speckle model is Rayleigh or normalized Fisher-Tippett distributed. Using speckle modeling based on these distributions, a speckle-specific information-theoretic feature detector is constructed and applied to provide feature images. Those feature images are then registered using differential equations, the solution of which provides a transformation to bring the images into alignment. Compared to standard gradient-based techniques, the experimental results demonstrate the effectiveness of our method, particularly for low contrast ultrasound images.

**Index Terms**—Speckle Image, Image registration, information theory, biomedical image processing

## I. INTRODUCTION

Within the current clinical setting, medical image registration is a crucial component of a large number of applications, including disease detection, analysis, and treatment. Image registration seeks to bring two or more images of the same scene into alignment.

Medical ultrasound images are a very important diagnostic tool for physicians. Ultrasound has many advantages, as it is fast, portable, cheap and safe. However, speckle noise degrades severely the quality of medical B-mode ultrasonic images, and renders the registration of ultrasound speckled images as a difficult problem. Speckle is an interference pattern resulting from the coherent accumulation of random scattering in a resolution cell of the ultrasound beam.

By concealing fine structures, the speckle has a detrimental effect to the current image registration algorithms, especially those based on comparing images on a pixel-to-pixel basis, since for ultrasound images, two corresponding pixels can have very different intensity levels due to the speckle.

In recent years, many registration algorithms have been proposed. However, few of them are designed specifically

for ultrasound speckle images [1]. Ultrasound-specific registration algorithms are presented in [2], [3]. They are based on pixel-to-pixel comparison between images. When the displacement is large, or images are taken of the same region from different scans, or different transducers, etc., these methods face difficulty, since the correlation of the speckle no longer exists. Unlike such previous work, our method relies on distribution-to-distribution comparisons. Consequently, it is significantly more robust.

The fully formed speckle is known to be a multiplicative Rayleigh distribution in the envelope detected image and Fisher-Tippett (doubly exponential) distribution in the log-compressed image. In our previous work, we use symmetrized Kullback-Liebler distance (J-divergence) of Fisher-Tippett distributed variables for comparing regions in a log-compressed ultrasound image in the context of feature detection [4]. However, since intensity of speckle is always non-negative, normalized Fisher-Tippett distribution is more suitable for speckle image modeling. Furthermore, some speckle images are not log-compressed and thus, Fisher-Tippett can no longer be applied. This paper expands our previous work to the cases of Rayleigh and normalized Fisher-Tippett distributed speckle images.

## II. STATISTICAL MODELING OF SPECKLE

### A. Rayleigh Case

Fully formed speckle is typically assumed when the number of scatters per cell is greater than ten [5]. It has been shown that the speckle in inphase/quadrature image,  $Q_I(x, y)$ , has a complex Gaussian distribution,

$$p(Q_I) = \frac{1}{2\pi\sigma^2} e^{-|Q_I(x,y)|^2/(2\sigma^2)} \quad (1)$$

Since  $Q_I$  is complex, its magnitude is taken to produce a real image. The distribution in this magnitude image  $I(x, y)$  is Rayleigh [6],

$$p(I) = \frac{I(x,y)}{\sigma^2} e^{-I(x,y)^2/2\sigma^2} \quad (2)$$

where  $I(x,y)$  is real. To obtain the maximum likelihood Rayleigh estimator, we differentiate the log likelihood of Eq.(2), with respect to  $\sigma$ , and set this expression to zero to determine the maximum likelihood estimate (MLE) of  $\sigma^2$ ,

$$\sigma^2 = \frac{\int_{\Omega} I(x,y)^2 dx dy}{2 \int_{\Omega} dx dy} \quad (3)$$

With the above equation, we can calculate the MLE of  $\sigma^2$  from the image intensities in the region assuming Rayleigh distribution. Note that this distribution is fully described by this one parameter.

### B. Normalized Fisher-Tippett Case

Let  $I(x,y)$  denote a pixel intensity in the decorrelated log magnitude IQ image at the location  $(x,y)$ . The normalized Fisher-Tippett PDF for a pixel's intensity can be written as

$$p(I(x,y)) = 2e^{\frac{1}{2\sigma^2}} e^{\left(2I(x,y) - \ln 2\sigma^2 - e^{2I(x,y) - \ln 2\sigma^2}\right)} \quad (4)$$

where  $\sigma^2$  denotes the normalized Fisher-Tippett parameter of the reflectivity samples. For a region  $\Omega$  in the image, the log likelihood can then be expressed as

$$\ell = \int_{\Omega} \left( \ln 2 + \frac{1}{2\sigma^2} + 2I(x,y) - \ln 2\sigma^2 - e^{2I(x,y) - \ln 2\sigma^2} \right) dx dy. \quad (5)$$

Next, we find an expression for  $\sigma^2$  that is the maximum likelihood estimator of the normalized Fisher-Tippett distribution, by taking the derivative of  $\ell$  and setting the expression equal to zero,

$$\frac{\partial \ell}{\partial \sigma} = \int_{\Omega} \left( -\frac{1}{\sigma^3} - \frac{4\sigma}{2\sigma^2} + \left( e^{2I(x,y) - \ln 2\sigma^2} \right) \frac{4\sigma}{2\sigma^2} \right) dx dy = 0. \quad (6)$$

We solve for  $\sigma^2$  as

$$\sigma^2 = \frac{\int_{\Omega} (e^{2I(x,y)} - 1) dx dy}{2 \int_{\Omega} dx dy} \quad (7)$$

Thus, given a region  $\Omega$  with area given by  $\int_{\Omega} dx dy$ , we can compute the maximum likelihood value of the normalized Fisher-Tippett distribution from the image intensities in the region.

## III. TWO-STAGE APPROACH

Our scheme consists of two steps, which are fully described in [4]. We give a short introduction in this section for self-completeness. For each image to be registered, we first apply our information-theoretic feature detector to the images and for each image compute an edge map which is robust to noise but still captures the significant edges in the image. Afterwards, we register these feature maps using a sum of squared differences (SSD) similarity metric, which is used to guide differential equations that update the registration.

### A. Feature detection

The feature detector we employ is based on a statistical comparison of regions in a speckle image [7]. As mentioned above, fully formed speckle in the magnitude image can be modeled using a Rayleigh or normalized Fisher-Tippett distribution. The Rayleigh distribution has the form in Eq.(2), where  $\sigma^2$  denotes the Rayleigh parameter of the reflectivity samples, while normalized Fisher-Tippett distribution is described in Eq.(4) and  $\sigma^2$  fully describes the distribution.

Given a region  $\Omega$  inside an ultrasound image, we can statistically estimate the Rayleigh distribution using the maximum likelihood estimator as in Eq.(3) and normalized Fisher-Tippett distribution using Eq.(7).

Our feature detector is based on information-theoretic comparison of two regions in an ultrasound image. That is, given two Rayleigh or normalized Fisher-Tippett distributions coming from different regions in the image, one parameterized by  $\sigma_1$  and the other by  $\sigma_2$ , we compute the J-divergence, or symmetrized Kullback-Liebler distance, as a measure of the degree of difference between the distributions. The J-divergence of two Rayleigh distributed variables was derived in [7] as

$$J = -1 + \frac{\sigma_1^2}{2\sigma_2^2} + \frac{\sigma_2^2}{2\sigma_1^2} \quad (8)$$

As stated above, the normalized Fisher-Tippett distribution models the intensities of fully formed speckle in the log magnitude image, which is the image that is typically presented to the user of an ultrasound machine. The Fisher-Tippett distribution is given by Eq.(4). Next, we derive an analytic expression for the Kullback-Liebler distance of two regions described by Fisher-Tippett distributions, as

$$D(p||q) = \int p \ln \frac{p}{q} dI \quad (9)$$

It can be simplified as

$$D(p||q) = \ln \left( \frac{\sigma_2^2}{\sigma_1^2} \right) - 1 + \frac{\sigma_1^2}{\sigma_2^2}. \quad (10)$$

Thus, the J-divergence of two Fisher-Tippett distributed variables is then

$$J = -1 + \frac{\sigma_1^2}{2\sigma_2^2} + \frac{\sigma_2^2}{2\sigma_1^2}. \quad (11)$$

Note that this is exactly the same expression for the J-divergence of two Rayleigh distributed variables. Eq.(11) is a similarity metric for ultrasound image registration of log magnitude IQ images.

Our feature detector has two sliding windows  $w_1$  and  $w_2$ . They are placed on either side of a pixel. Given the set of pixels in  $w_1$ , Rayleigh or normalized Fisher-Tippett parameter  $\sigma_1^2$  and  $\sigma_2^2$  are determined using Eq.(3) or Eq.(7). Then, we compute J-divergence between these two distributions using Eq.(8) or Eq.(11) as a measure of how different the regions are. When the windows are placed to the left and to the right of the pixel, this gives a horizontal distance map  $J_x(x,y)$  that is functionally similar to the gradient operator in the  $x$

direction, except that the values are non-negative. This can be repeated in the  $y$  direction. Here, we define a feature map  $F(x,y)$  as

$$F(x,y) = \sqrt{J_x(x,y)^2 + J_y(x,y)^2}. \quad (12)$$

This feature detector is much less distracted by the speckle compared to the gradient estimator, yet still detects the important edges in the image.

For each image to be registered, it is applied to transform the image into a feature-detected image that contains the important edges needed for registration while simultaneously mitigating false responses due to the speckle. These feature-detected images are then passed to the registration algorithm, to be described next.

### B. Registration

Let  $T(x,y)$  be the transformation between the two feature detected images,  $F_1$  and  $F_2$ . Our goal is to estimate the parameters of the transformation so that the feature images become aligned. To accomplish this, we minimize an energy function based on the sum of square differences between two feature maps,

$$E(T(x,y)) = \int [F_1(x,y) - F_2(T(x,y))]^2 dx dy, \quad (13)$$

where the transformation is applied to the second image. Starting with an initial guess, we can iteratively update the transformation using PDEs based on a Gauss-Newton optimization [8] to minimize the energy functional in Eq.(13). Upon convergence, the transformation is a local optimum of the energy.

## IV. EXPERIMENTAL RESULTS

Although there do exist speckled images which are not log-compressed, display ultrasound images usually go through a log-compression process. For completeness, we discuss the Rayleigh case in the above section. However, the experiments mainly focus on normalized Fisher-Tippett distribution.

In the experiments, we generate synthetic speckle images to study the registration performance as the image contrast is diminished. There are two images at each contrast level. The ground truth registration parameters are (5,5) for the translation and  $5^\circ$  for the rotation. The images of normalized Fisher-Tippett distribution and their feature detection results are shown in Fig.2. For comparison, a standard edge map formed with a difference of Gaussian filter is also created. It is obvious that the normalized Fisher-Tippett feature detector produces cleaner edge as it robustly identifies the important features without many false detections due to speckle noise.

Fig.1 shows the registration error, both in translation and rotation. It is denoted as the squared error of the estimated parameter compared to the ground truth value. We can easily notice that the registration error of the gradient-based edge maps (solid blue curves) quickly increases as the contrast is diminished, while the registration of the proposed method

(dashed red curve) is significantly lower. It is similar to Fisher-Tippett case in [4].

For the region image extracted from an abdominal part ultrasound image, we applied our method again to examine its effectiveness, depicted in Fig.3. Since there is no ground truth, we compute the sum of squared differences (SSD) between the original and registered images. The SSD decreased by 51.14% for the proposed method while it decreased only by 12.73% with the standard gradient scheme.

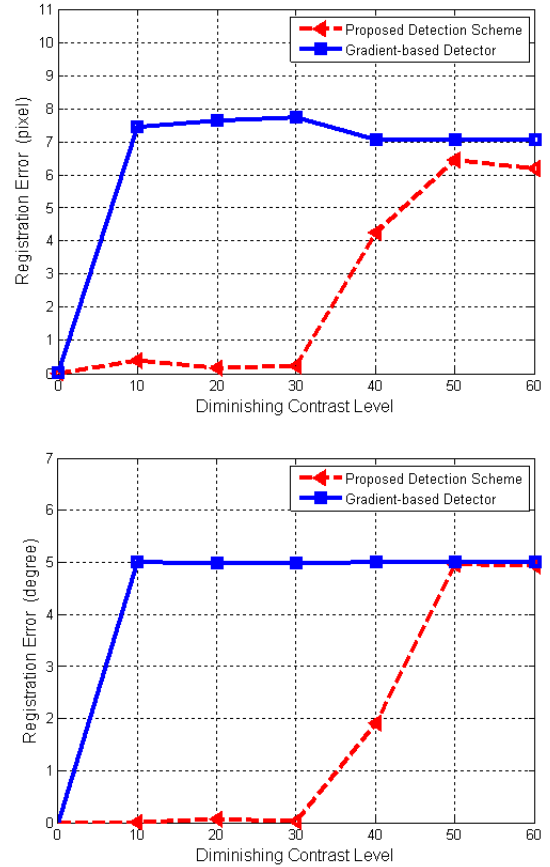


Fig. 1. Registration error as a function of diminishing contrast. The top figure shows the translational error while the bottom one shows rotational error.

## V. CONCLUSION

In this article, we use speckle statistical model based on normalized Fisher-Tippett distribution to construct a speckle-specific information-theoretic feature detector and utilize these feature maps for speckle image registration. Since the similarity metric is distribution to distribution comparison, rather than pixel-pixel intensity comparison, it is more robust to uncorrelated speckle noise. Our experiment results also demonstrate the effectiveness of this method.

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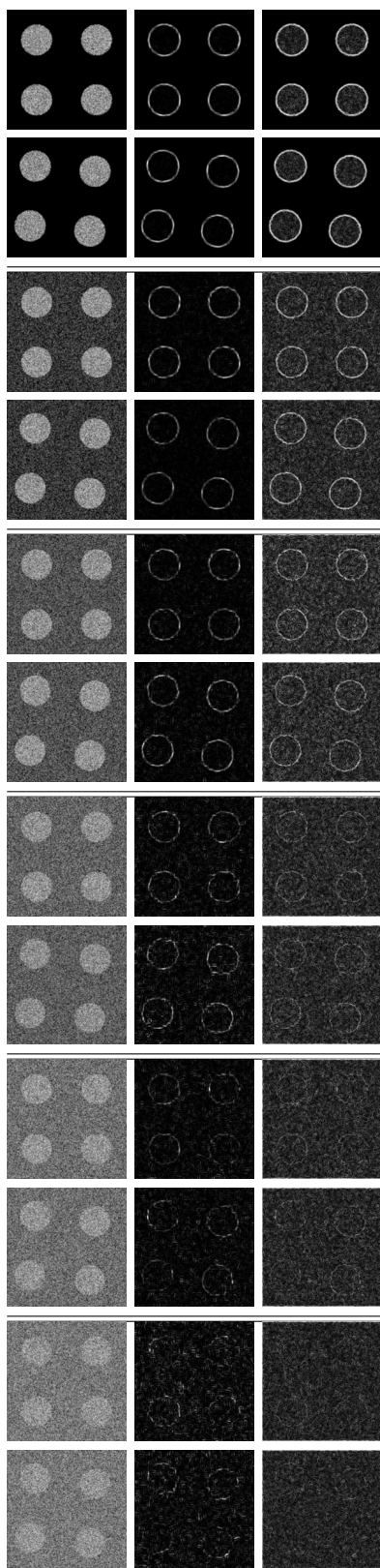


Fig. 2. Feature detection in synthetic ultrasound images. The leftmost column shows the original synthetic ultrasound images with diminishing contrast. The middle column shows the feature detection with information-theoretic method, while the rightmost column shows the feature detection with the difference of Gaussian filter.

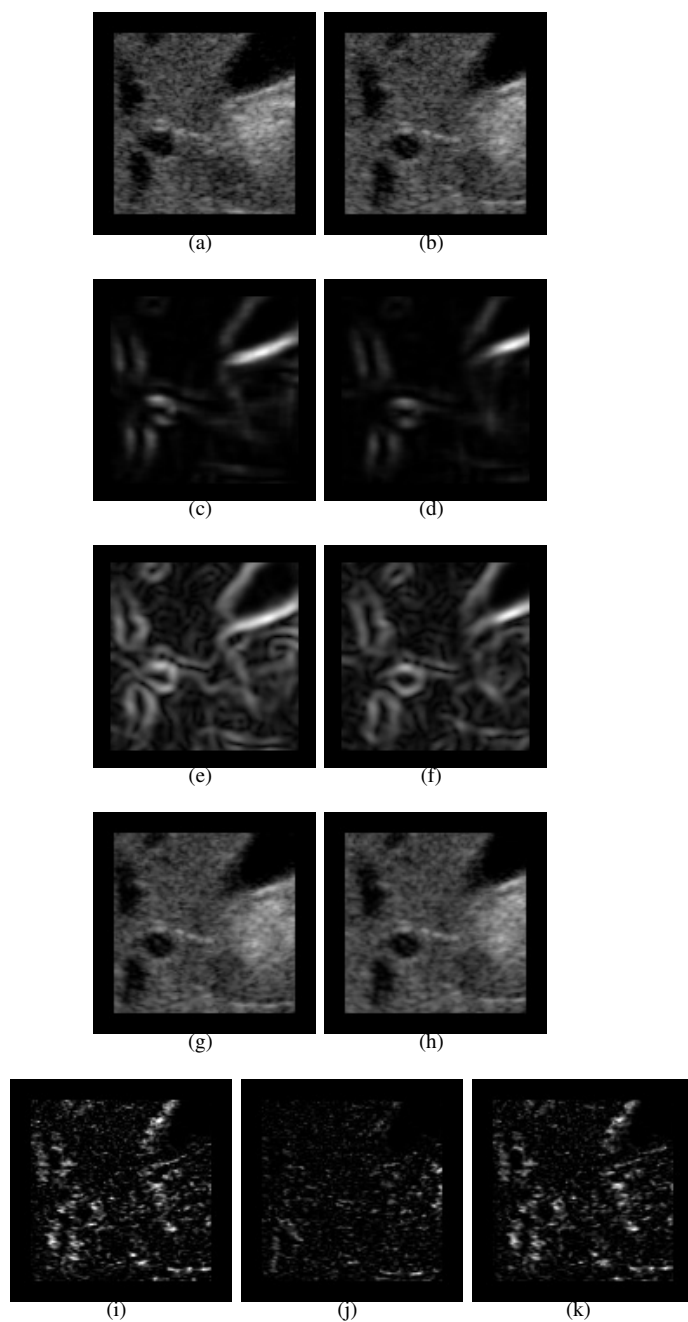


Fig. 3. Registration results of ultrasound images. Original (a)(b), feature detection with Fisher-Tippett (c)(d), gradient-based feature detection (e)(f), registered image with Fisher-Tippett and gradient detector (g)(h), original residue image (i), residue image with Fisher-Tippett detector (j), residue image with gradient detector (k).

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