How to explain an increasing proportion of people testing positive for COVID if there is neither an increase in proportion of genuine cases nor increase in the false positive rate

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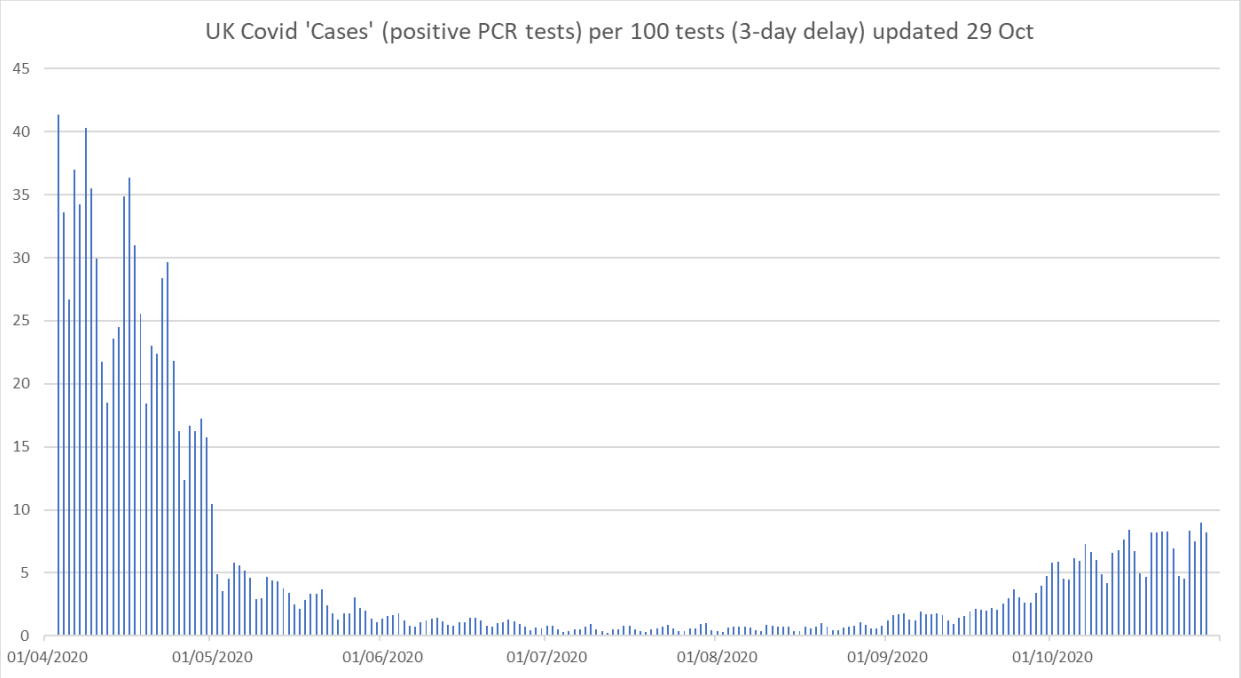
A few people have argued[1] that, despite the official Government statistics, real COVID cases are not increasing and that most COVID positive test results are false positives. The obviously strong argument against this is the evidence that the ***proportion*** of positive tests is increasing as shown here:



Ignoring the possibility that COVID cases are being redefined, then unless the false positive rate itself is increasing, surely it cannot be possible for the proportion of positive tests to be increasing without a proportional increase in genuine COVID cases? In fact, as we have pointed out many times[2], there could be simple causal explanations for an increasing proportion of positive tests being observed even if the underlying COVID infection rate and test accuracy rates are unchanged. The most obvious causal explanation would be a change in the type of people being tested, such as if a lower proportion of people without symptoms were tested, or a higher proportion of older people were tested. In other words, if a higher proportion of people who had the virus were being tested.

However, as has been pointed out by some[3], it ***is*** possible for the proportion of ***people*** wrongly testing positive (as opposed to the proportion of tests that are wrongly positive) to increase even if there is also no change to the type of people being tested. How? Because if some people are being tested more than once – as is certainly happening for those admitted to

[1] https://lockdownsceptics.org/lies-damned-lies-and-health-statistics-the-deadly-danger-of-false-positives/ https://spectator.us/covid-19-false-positive-trap-seasonal/

[2] https://probabilityandlaw.blogspot.com/2020/10/why-we-know-so-little-about-covid-19.html

[3] https://twitter.com/ClareCraigPath/status/1312629163036155909?s=20

hospital – the probability of those people wrongly testing positive at least once ***does*** increase. Suppose, for example, the 'constant' false positive rate is 1 in 100 (i.e. 0.01 probability). Then, if you have two tests, the probability you test positive at least once is higher than 0.01. In fact, it is almost double. It is one minus the probability you do NOT test positive both times, i.e.

$$1 - (0.99) \times (0.99) = 1 - 0.9801 = 0.01999$$

which is close to 2%. But we do not even need to use this calculation to show the impact of testing people more than once.

Consider the following example:

> Suppose the real COVID infection rate among those being tested is constant at 5% and that 10,000 people in consecutive periods are tested. Suppose the true positive rate for the testing is 80% and the false positive rate is 1%.
>
> In period 1 everybody is tested just once. Then about 500 of these people have COVID and 9500 do not. But then 95 of the 9500 without COVID will wrongly test positive. Hence 495 out of 10000 people test positive – i.e. a proportion of 4.95%
>
> If the rates are unchanged in subsequent periods, then the proportion testing positive remains the same no matter how many more (or less) are tested.
>
> However, suppose that in period 2 some of those – let's say 20% - who test negative are retested and are recorded as a positive case if the second test is positive. Then, if 10,000 are tested in period 2, we know from above that 495 will test positive first time. However, 20% of those who test negative are retested. That means 20 of the 100 who wrongly tested negative and 1881 of those 9405 who correctly tested negative will be retested. Of the 20 with COVID, about 16 will test positive. Of the 1881 without COVID about 19 will test positive. Hence, in period 2 – because some get tested twice – the number testing positive is now 529 out of 10,000 – i.e. a proportion of 5.29%.

It follows that if, over a sequence of periods, a steadily increasing proportion of people get retested, then the proportion of **people** testing positive will also steadily increase even though the true infection rate and false positive rate remain constant and there is no change in the type of people being tested. Moreover, if some people get retested more than twice (as is happening with hospital admissions) then this leads to further increases in the proportion testing positive.

It should be noted that an increase in the true positive rate (i.e. a reduction in the false negative rate) will ***also*** lead to an increase in the proportion of positive tests even if the true infection rate and false positive rate are unchanged and nobody gets retested. In fact, this is a point which has largely been missed in the discussion about false positives for PCR testing. At the start of the crisis false negatives rates were typically as high as 67%[4], and it would be understandable in such cases to both a) do retesting and b) increase the sensitivity of the tests to reduce the false negative rate. The sensitivity can be increased by increasing the cycle threshold of the PCR test. However, it is this increase which has led to claims of the reduction in specificity (the proportion of true negatives), i.e. an increase in the false positive rate.

It is widely assumed that the false negative rates ***have*** been significantly reduced. This would normally lead to an ***increase*** in the false positive rate. But even if, somehow, the false positive rate also remained the same, there would still be an increasing proportion of people testing

[4] https://www.dailymail.co.uk/health/article-8406621/Coronavirus-tests-return-false-negatives-67-time-theyre-given-four-days-infection.html

positive if there was no change in the infection rate and even if we ignore retesting and just consider proportion of tests that are positive (and not proportion of people who test positive). To see this, consider a similar example as above:

> Period 1 is exactly as above (5% real COVID infection rate among those being tested is constant at 5% a true positive rate of 80% and false positive rate of 1%), so the proportion of positive tests is 4.95%.
>
> Suppose the only thing that changes in period 2 is an increase in the true positive rate to 90%. Then this time 450 (as opposed to 400) of those who have COVID test positive making a total of 545 testing positive, so the proportion of positive tests is now increased to 5.45%.

So, in summary, even if there are no changes to the way COVID cases are defined, then there are still several different ways in which we could see an increasing proportion of people testing positive even if the underlying COVID infection rate, and the false positive rate, are unchanged, namely:

1. Causal explanation, such as change in the type of people being tested
2. Increasing number of people being retested
3. Increasing true positive rate for the testing

Obviously combining any of the above will lead to even greater increases